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## METHODS AND TOOLS FOR IDENTIFICATION OF NONLINEAR DYNAMICAL SYSTEMS BASED ON VOLTERRA MODELS IN FREQUENCY DOMAIN

*The accuracy and computational stability of deterministic identification methods for nonlinear dynamical systems in the form of multidimensional frequency characteristics of amplitude and phase are investigated. Approximation and interpolation methods of identification with use of test polyharmonic signals are considered. The computational stability of the identification procedure is provided by the method of regularization of ill-posed problems. Wavelet filtering is used to smooth the estimates of the obtained characteristics.*

**Key words:** nonlinear dynamic systems, nonparametric identification, Volterra model, multidimensional frequency characteristics, polyharmonic signals, regularization, wavelet filtering.

**Problem statement.** The sensors in remote sensing systems are looking through a layer of atmosphere separating the sensors from the Earth’s surface being observed. It is essential to understand the effects of atmosphere on the electromagnetic radiation travelling from the Earth to the sensor through the atmosphere. The atmospheric constituents cause wavelength dependent absorption and scattering of radiation due to environment interactions, emissions and so on (Fig. 1) [1]. The same problems arise in the study of coastal shelves associated with underwater monitoring, as well as with the development of underwater mineral deposits. Ultrasonic signals and measuring devices – echo-sounders (Fig. 2) are used to measure depths, display the profile and approximate structure of the bottom, search and classify various objects at the bottom and in the water column, as well as to perform various navigation tasks [2]. The atmosphere and water environment between radiating surface and sensor can be understood as a wireless communication channel (CC). The technical conditions of CC during operation should be considered for the effective communications. Changes during data transfer can decrease the rate of data transmission in digital CC up to stop of transmission. In analog CC it can be cause distortions and noise of the transmitted signals. Some of the atmospheric effects can be corrected before the sensing data is subjected to further analysis and interpretation. These effects degrade the adequateness of received data. The new methods and supporting tools are developed to automate the measurement and consideration of the characteris-

tics of the CC. It helps to build the information and mathematical models of nonlinear dynamic object such as the CC [3–5], i. e. to solve the identification problem.

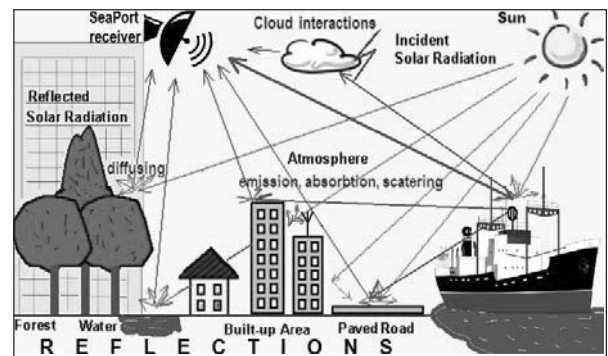


Fig. 1. Environment effects in remote sensing systems

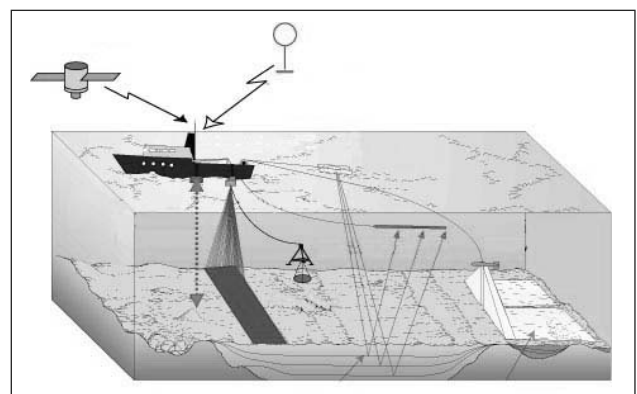


Fig. 2. Sonar monitoring of marine areas

Building Volterra models and using them for visualization for such complex nature effects as waves of

sea surface were well studied in [6–8]. This methodic allows building linear and nonlinear models for different systems. Modern continuous CCs are nonlinear stochastic inertial systems. The model in the form of integral Volterra series used to identify them [7; 9–16]. The nonlinear and dynamic properties of such system are completely characterized by a sequence of multidimensional weighting functions – Volterra kernels).

Building a model of nonlinear dynamic system in the form of a Volterra series lies in the choice of the test actions form. Also it uses the developed algorithm that allows determining the Volterra kernels and their Fourier-images for the measured responses (multidimensional amplitude–frequency characteristics (AFC) and phase–frequency characteristics (PFC)) to simulate the CC in the time or frequency domain, respectively [17–35].

The additional research of new method of nonlinear dynamical systems identification, based on the Volterra model in the frequency domain is proposed. This method lies in n-fold differentiation of responses of the identifiable system by the amplitude of the test polyharmonic signals. The developed identification toolkit is used to build information model of the test nonlinear dynamic system in the form of the first, second and third order model [36–47].

The aim of this work is to identify the continuous CC using Volterra model in the frequency domain, i. e. the determination of its multi-frequency characteristics on the basis of the data of the input-output experiment, using test polyharmonic signals and interpolation method to obtain model coefficients [21–23; 25; 41; 47; 48].

**1. Volterra models**

Generally, “input-output” type ratio for nonlinear dynamical system can be presented by Volterra series [10; 49]:

$$y[x(t)] = w_0(t) + \int_0^{\infty} w_1(\tau)x(t-\tau)d\tau + \int_0^{\infty} \int_0^{\infty} w_2(\tau_1, \tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} w_3(\tau_1, \tau_2, \tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \dots = w_0(t) + \sum_{n=1}^{\infty} y_n[x(t)] \tag{1}$$

where the n-th partial component of response of the system is

$$y_n[x(t)] = \int_0^t \dots \int_0^t w_n(\tau_1, \dots, \tau_n) \prod_{i=1}^n x(t-\tau_i) d\tau_i, \tag{2}$$

x(t) and y(t) are input and output signals of system respectively; w<sub>n</sub>(τ<sub>1</sub>, ..., τ<sub>n</sub>) – weight function or

n-order Volterra kernel; y<sub>n</sub>[x(t)] – n-th partial component of system’s response; w<sub>0</sub>(t) – denotes free component of the series (for zero initial conditions w<sub>0</sub>(t)=0); t – current time.

Commonly, the Volterra series are replaced by a polynomial, with only taking several first terms of series (1) into consideration. Nonlinear dynamical system identification in a form of Volterra series consists in n-dimensional weighting functions determination w<sub>n</sub>(τ<sub>1</sub>, ..., τ<sub>n</sub>) for time domain or it’s Fourier transforms W<sub>n</sub>(jω<sub>1</sub>, ..., jω<sub>n</sub>) – n-dimensional transfer functions for frequency domain.

Multidimensional Fourier transform for n-order Volterra kernel (1) is written in a form:

$$W_n(j\omega_1, \dots, j\omega_n) = F_n \langle w_n(\tau_1, \dots, \tau_n) \rangle = \int_0^{\infty} \dots \int_0^{\infty} w_n(\tau_1, \dots, \tau_n) \exp\left(-j \sum_{i=1}^n \omega_i \tau_i\right) \prod_{i=1}^n d\tau_i, \tag{3}$$

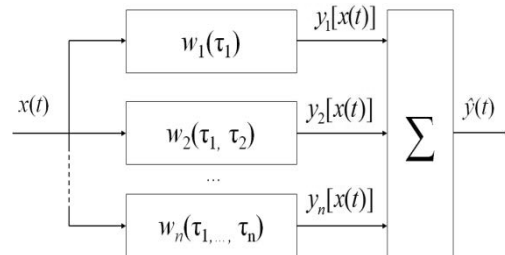
where F<sub>n</sub>⟨⟩ – n-dimensional Fourier transform; j = √-1.

Then the model of nonlinear system based on Volterra model in frequency domain can be represented as:

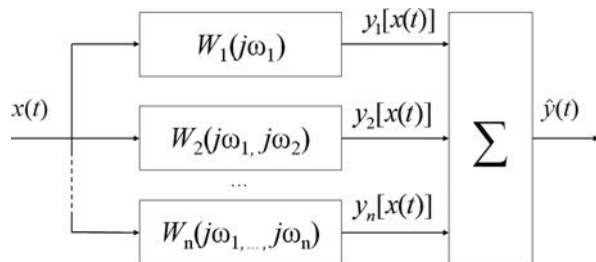
$$y[x(t)] = \sum_{n=1}^{\infty} F_n^{-1} \left\langle W_n(j\omega_1, \dots, j\omega_n) \prod_{i=1}^n X(j\omega_i) \right\rangle, \tag{4}$$

where F<sub>n</sub><sup>-1</sup>⟨⟩ – inverse n-dimensional Fourier transform, it is used here under the condition of variables association t<sub>1</sub>=...=t<sub>n</sub>=t; X(jω<sub>i</sub>) – Fourier transform of input signal.

Volterra model structured charts in the time and frequency domain is shown in Fig. 3 and Fig. 4.



**Fig 3. Volterra model structured chart in the time domain**



**Fig 4. Volterra model structured chart in the frequency domain**



Table 2

Amplitudes and corresponding coefficients of the interpolation method

$n$	$N$	$a_1^{(n)}$	$a_2^{(n)}$	$a_3^{(n)}$	$a_4^{(n)}$	$a_5^{(n)}$	$a_6^{(n)}$	$c_1^{(n)}$	$c_2^{(n)}$	$c_3^{(n)}$	$c_4^{(n)}$	$c_5^{(n)}$	$c_6^{(n)}$
1	2	-1	1					-0,5	0,5				
	4	-1	-0,5	0,5	1			0,0833	-0,6667	0,6667	-0,0833		
	6	-1	-0,67	-0,33	0,33	0,67	1	-0,0167	0,15	-0,75	0,75	-0,15	0,0167
2	2	-1	1					1	1				
	4	-1	-0,5	0,5	1			-0,0833	1,3333	1,3333	-0,0833		
	6	-1	-0,67	-0,33	0,33	0,67	1	0,0111	-0,15	1,5	1,5	-0,15	0,0111
3	4	-1	-0,5	0,5	1			-0,5	1	-1	0,5		
	6	-1	-0,67	-0,33	0,33	0,67	1	0,125	-1	1,625	-1,625	1	-0,125

Volterra series is offered [36–46]. It is used  $n$ -fold differentiation of a target signal on parameter-amplitude  $a$  of test actions to separate the response of the nonlinear dynamical system on partial components  $y_n[x(t)]$ .

*Affirmation 1.* Let at input of system test signal of  $ax(t)$  kind is given, where  $x(t)$  – is arbitrary function and  $a$  – is scale coefficient (amplitude of signal), where  $0 < |a| \leq 1$ , then for the selection of a partial component of the  $n$ -th order  $\hat{y}_n(t)$  from measurement of the response nonlinear system  $y[ax(t)]$  in the form of Volterra series, it is necessary to determine  $n$ -th partial derivative of the total response amplitude  $a$  where  $a=0$

$$n! \int_0^\infty \dots \int_0^\infty w_n(\tau_1, \dots, \tau_n) \prod_{k=1}^n x(t - \tau_k) d\tau_k = y_a^{(n)} [a x(t)]_{a=0} \quad (10)$$

We use the method of extracting the partial components with the help of  $n$ -fold differentiation of the response  $y[ax(t)]$  with respect to parameter – amplitude  $a$  and the use of the derivative value at  $a=0$  [36–38].

Injecting an input signal  $ax(t)$  where  $a$  is the scaling factor (signal amplitude), one has the following response of the nonlinear system:

$$y[a \cdot x(t)] = a \int_0^\infty w(\tau) \cdot x(t - \tau) d\tau + a^2 \int_0^\infty \int_0^\infty w_2(\tau_1, \tau_2) x(t - \tau_1) x(t - \tau_2) d\tau_1 d\tau_2 + a^n \int_0^\infty \dots \int_0^\infty w_n(\tau_1, \dots, \tau_n) \prod_{r=1}^n x(t - \tau_r) d\tau_r + \dots \quad (11)$$

To distinguish the partial component of the  $n$ -th order, differentiate the system response  $n$  times with respect to the amplitude:

$$\frac{\partial^n y[a \cdot x(t)]}{\partial a^n} = n! \int_0^\infty \dots \int_0^\infty w_n(\tau_1, \dots, \tau_n) \prod_{r=1}^n x(t - \tau_r) d\tau_r + (n+1)! \cdot a \int_0^\infty \dots \int_0^\infty w_{n+1}(\tau_1, \dots, \tau_{n+1}) \prod_{r=1}^{n+1} x(t - \tau_r) d\tau_r + \dots \quad (12)$$

Taking the value of the derivative at  $a=0$ , we finally obtain the expression for the partial component (7).

*Formulas numerical differentiation.* Partial derivative should be substituted by form of finite difference for calculation. Differentiation of function, which was set in discrete points, could be accomplished by means of numerical computing after preliminary smoothing of measured results. Various formulas for the numerical differentiation are known, which differ from each other by means of error.

Let's use universal reception which allows to substitute a derivative of any  $n$  order for differential ratio so that the error from such replacement for function  $y(a)$  was any beforehand set order of  $p$  approximation concerning a step of  $h=\Delta a$  of computational mesh on amplitude. Method of undetermined coefficient for equality [50]

$$\frac{d^n y(a)}{da^n} = \frac{1}{h^n} \sum_{r=-r_1}^{r_2} c_r y(a+rh) + O(h^p), \quad (13)$$

where the coefficients  $c_r$  are taken not depending on  $h$ ,  $r = -r_1, -r_1 + 1, \dots, -1, 0, 1, \dots, r_2 - 1, r_2$ , so that equality (13) was fair. Limits of summation  $r_1 \geq 0$  и  $r_2 \geq 0$  could be arbitrary, but so that the differential relation  $h^{-n} \sum c_r y(a+rh)$  of  $r_1 + r_2$  order have to satisfy to inequality  $r_1 + r_2 \geq n + p - 1$ .

To define the  $c_r$  it is necessary to solve the following set of equations

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ -r_1 & -r_1 + 1 & \dots & r_2 \\ \dots & \dots & \dots & \dots \\ (-r_1)^{n-1} & (-r_1 + 1)^{n-1} & \dots & r_2^{n-1} \\ (-r_1)^n & (-r_1 + 1)^n & \dots & r_2^n \\ (-r_1)^{n+1} & (-r_1 + 1)^{n+1} & \dots & r_2^{n+1} \\ \dots & \dots & \dots & \dots \\ (-r_1)^{n+p-1} & (-r_1 + 1)^{n+p-1} & \dots & r_2^{n+p-1} \end{bmatrix} \cdot \begin{bmatrix} c_{-r_1} \\ c_{-r_1+1} \\ \dots \\ c_{-1} \\ c_0 \\ c_1 \\ \dots \\ c_{r_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ n \\ 0 \\ \dots \\ 0 \end{bmatrix} \quad (14)$$

If  $r_1 + r_2 = n + p - 1$ , then inscribed in  $n + p$  equality forms linear system concerning the same number of  $c_r$  unknown. The determiner of this system is Vandermonde's determiner and differs from zero. Thus, there is only one set of  $n$  coefficients, satisfying the system.

If  $r_1 + r_2 \geq n + p$ , then there are many such sets of coefficients  $c_r$ .

On the basis of (13) in [40] the formulas of derivative calculation of the first, second and third orders are received at  $a=0$  with use of the central differences for equidistant nodes of the computational grid.

In work formulas for numerical differentiation with use of the central differences for equidistant assembly are used. Volterra kernel of the first order is determined by formulas as the first derivative at  $r_1 = r_2 = 1$ ,  $r_1 = r_2 = 2$  or  $r_1 = r_2 = 3$  respectively

$$\begin{aligned} y'_0 &= \frac{1}{2h}(-y_{-1} + y_1), \\ y'_0 &= \frac{1}{12h}(y_{-2} - 8y_{-1} + 8y_1 - y_2), \\ y'_0 &= \frac{1}{60h}(-y_{-3} + 9y_{-2} - 45y_{-1} + 45y_1 - 9y_2 + y_3). \end{aligned} \quad (15)$$

Volterra kernel of the first order is determined by formulas as the first derivative at  $r_1 = r_2 = 1$ ,  $r_1 = r_2 = 2$  or  $r_1 = r_2 = 3$  respectively

$$\begin{aligned} y''_0 &= \frac{1}{h^2}(y_{-1} - 2y_0 + y_1), \\ y''_0 &= \frac{1}{12h^2}(-y_{-2} + 16y_{-1} - 30y_0 + 16y_1 - y_2), \\ y''_0 &= \frac{1}{180h^2}(2y_{-3} - 27y_{-2} + 270y_{-1} - 490y_0 + 270y_1 - 27y_2 + 2y_3). \end{aligned} \quad (16)$$

Volterra kernel of the first order is determined by formulas as the first derivative at  $r_1 = r_2 = 2$  or  $r_1 = r_2 = 3$  respectively

$$\begin{aligned} y'''_0 &= \frac{1}{2h^3}(-y_{-2} + 2y_{-1} - 2y_1 + y_2), \\ y'''_0 &= \frac{1}{8h^3}(y_{-3} - 8y_{-2} + 13y_{-1} - 13y_1 + 8y_2 - y_3). \end{aligned} \quad (17)$$

In the formulas written above, we use the following notations

$$y'_0 = y'(0), y''_0 = y''(0), y'''_0 = y'''(0); y_r = y(rh), r = 0, \pm 1, \pm 2; \pm 3,$$

where we put  $y_0=0$ , since identification nonlinear systems is implemented with zero initial conditions.

The amplitudes of the test signals  $a_i^{(n)}$  and the corresponding coefficients  $c_i^{(n)}$  for responses are shown in Table 2, where  $n$  – order of the estimated Volterra kernel;  $i$  – number of the experiment ( $i=1,2,\dots,N$ ), where  $N=r_1+r_2$ , i.e. number of interpolation knots (number of experiments).

**2.3. An approximation model.** Is developing a method of constructing approximate Volterra model of the NDS [51]. Method identification is based on the approximation  $y(t)$  at an arbitrary deterministic signal  $x(t)$  in the form of integral power of the polynomial Volterra  $N$ -th order ( $N$  – order approximation model)

$$\tilde{y}_N(t) = \sum_{n=1}^N \hat{y}_n(t) = \sum_{n=1}^N \int_0^{\infty} \dots \int_0^{\infty} w_n(\tau_1, \dots, \tau_n) \prod_{i=1}^n x(t - \tau_i) d\tau_i. \quad (18)$$

*Affirmation 2.* Let the input test signals NDS are fed alternately  $a_1x(t), a_2x(t), \dots, a_Lx(t); a_1, a_2, \dots, a_L$  –

distinct real numbers satisfying the condition  $|a_j| \leq 1$  for  $\forall j=1,2,\dots,L$ ; then

$$\begin{aligned} \tilde{y}_N[a_jx(t)] &= \sum_{n=1}^N \hat{y}_n[a_jx(t)] = \\ &= \sum_{n=1}^N a_j^n \int_0^{\infty} \dots \int_0^{\infty} w_n(\tau_1, \dots, \tau_n) \prod_{i=1}^n x(t - \tau_i) d\tau_i = \sum_{n=1}^N a_j^n \hat{y}_n(t) \end{aligned} \quad (19)$$

The partial components in the approximation model  $\hat{y}_n(t)$  are found using the least square method (LSM). This makes it possible to obtain such evaluation in which the sum of squared deviations of responses identified the nonlinear dynamical system  $y[a_jx(t)]$  on the model  $\hat{y}_N[a_jx(t)]$  response is minimal, i.e., NDS provides a minimum criterion

$$\begin{aligned} J_N &= \sum_{j=1}^L (y[a_jx(t)] - \tilde{y}_N[a_jx(t)])^2 = \\ &= \sum_{j=1}^L \left( y_j(t) - \sum_{n=1}^N a_j^n \hat{y}_n(t) \right)^2 \rightarrow \min, \end{aligned} \quad (20)$$

where  $y_j(t) = y[a_jx(t)]$ . Minimization of the criterion (20) is reduced to solving the system of normal equations of Gauss, which in vector-matrix form can be written as

$$A'A\hat{y} = A'\bar{y}, \quad (21)$$

where

$$A = \begin{bmatrix} a_1 & a_1^2 & \dots & a_1^N \\ a_2 & a_2^2 & \dots & a_2^N \\ \dots & \dots & \dots & \dots \\ a_L & a_L^2 & \dots & a_L^N \end{bmatrix}, \bar{y} = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \dots \\ y_L(t) \end{bmatrix}, \hat{y} = \begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \dots \\ \hat{y}_N(t) \end{bmatrix}.$$

From (21) we obtain

$$\hat{y} = (A'A)^{-1}A'\bar{y} \quad (22)$$

In (8), matrix operations, we obtain

$$\begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \dots \\ \hat{y}_N(t) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^L a_j^2 & \sum_{j=1}^L a_j^3 & \dots & \sum_{j=1}^L a_j^{N+1} \\ \sum_{j=1}^L a_j^3 & \sum_{j=1}^L a_j^4 & \dots & \sum_{j=1}^L a_j^{N+2} \\ \dots & \dots & \dots & \dots \\ \sum_{j=1}^L a_j^{N+1} & \sum_{j=1}^L a_j^{N+2} & \dots & \sum_{j=1}^L a_j^{2N} \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^L a_j y_j(t) \\ \sum_{j=1}^L a_j^2 y_j(t) \\ \dots \\ \sum_{j=1}^L a_j^N y_j(t) \end{bmatrix}. \quad (23)$$

**2.4. Using polyharmonic test signals.** The test polyharmonic effects for identification in the frequency domain representing by signals of such type:

$$x(t) = \sum_{k=1}^n A_k \cos(\omega_k t + \phi_k), \quad (24)$$

where  $n$  – the order of transfer function being estimated;  $A_k, \omega_k$  and  $\phi_k$  – accordingly amplitude, frequency and a phase of  $k$ -th harmonics. In research, it is supposed every amplitude of  $A_k$  to be equal, and phases  $\phi_k$  equal to zero.

For identification in the frequency domain the test polyharmonic signals are used. We prove:

*Affirmation 3.* If test polyharmonic signal is used in form

$$x(t) = A \sum_{k=1}^n \cos \omega_k t = \frac{A}{2} \sum_{k=1}^n (e^{j\omega_k t} + e^{-j\omega_k t}), \quad (25)$$

then the  $n$ -th partial component of the response of test system can be written in form:

$$y_n(t) = \frac{A^n}{2^{n-1}} \sum_{m=0}^{E(n/2)} C_n^m \sum_{k_1=1}^n \dots \sum_{k_n=1}^n |W_n(-j\omega_{k_1}, \dots, -j\omega_{k_m}, j\omega_{k_{m+1}}, \dots, j\omega_{k_n})| \times \cos \left( \left( -\sum_{l=0}^m \omega_{k_l} + \sum_{l=m+1}^n \omega_{k_l} \right) t + \arg W_n(-j\omega_{k_1}, \dots, -j\omega_{k_m}, j\omega_{k_{m+1}}, \dots, j\omega_{k_n}) \right), \quad (26)$$

where  $E()$ – function used to obtain the of integer part of the value [52–54].

The component with frequency  $\omega_1 + \dots + \omega_n$  is extracted from the response to test signal (26):

$$A^n |W_n(j\omega_1, \dots, j\omega_n)| \cos[(\omega_1 + \dots + \omega_n)t + \arg W_n(j\omega_1, \dots, j\omega_n)]. \quad (27)$$

Certain limitations should be imposed while choosing of frequency polyharmonic test signals in a process determining multidimensional AFC and PFC. This is the reason why the values of AFC and PFC in this unallowable points of multidimensional frequency space can be calculated using interpolation only. In practical realization of nonlinear dynamical systems identification it is needed to minimize number of such undefined points at the range of multidimensional frequency characteristics determination. This was performed to provide a minimum of restrictions on choice of frequency of the test signal. It is shown that existed limitation can be weakened. New limitations on choice of frequency are reducing number of undefined points.

After analyzing the (26) it is defined: to obtain Volterra kernels for nonlinear dynamical system in frequency domain the limitations on choice of frequencies of test polyharmonic signals have to be restricted. These restrictions provide inequality of combination frequencies in the test signal harmonics. The theorem about choice of test signals frequencies is proven.

*The theorem about choice of test signals frequencies.* For the definite filtering of a response of the harmonics with combination frequencies  $\omega_1 + \omega_2 + \dots + \omega_n$  within the  $n$ -th partial component it is necessary and sufficient to keep the frequency from being equal to another combination frequencies of type  $k_1\omega_1 + \dots + k_n\omega_n$ , where the coefficients  $\{k_i | i=1, 2, \dots, n\}$  must satisfy the conditions:

1) number  $K$  of negative value coefficients ( $k_i < 0$ ) is in  $0 \leq K \leq E(n/2)$  (where  $E$  – function used to obtain the of integer part of the value);

$$2) \sum_{i=1}^n |k_i| \leq n;$$

$$3) \sum_{i=1}^n |k_i| \equiv n \pmod{2}, \quad n - \sum_{i=1}^n |k_i| = 2l, \quad l \in \mathbb{N}.$$

It was shown that during determination of multi-dimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies. This provides inequality of combination frequencies in output signal harmonics:  $\omega_1 \neq 0$ ,  $\omega_2 \neq 0$  and  $\omega_1 \neq \omega_2$  for the second order identification procedure, and  $\omega_1 \neq 0$ ,  $\omega_2 \neq 0$ ,  $\omega_3 \neq 0$ ,  $\omega_1 \neq \omega_2$ ,  $\omega_1 \neq \omega_3$ ,  $\omega_2 \neq \omega_3$ ,  $2\omega_1 \neq \omega_2 + \omega_3$ ,  $2\omega_2 \neq \omega_1 + \omega_3$ ,  $2\omega_3 \neq \omega_1 + \omega_2$ ,  $2\omega_1 \neq \omega_2 - \omega_3$ ,  $2\omega_2 \neq \omega_1 - \omega_3$ ,  $2\omega_3 \neq \omega_1 - \omega_2$ ,  $2\omega_1 \neq -\omega_2 + \omega_3$ ,  $2\omega_2 \neq -\omega_1 + \omega_3$  and  $2\omega_3 \neq -\omega_1 + \omega_2$  for the third order identification procedure [52].

### 3. The techniques of test system identification

Described method was tested using nonlinear test system (fig. 5) represented by Riccati equation

$$\frac{dy(t)}{dt} + \alpha y(t) + \beta y^2(t) = u(t). \quad (28)$$

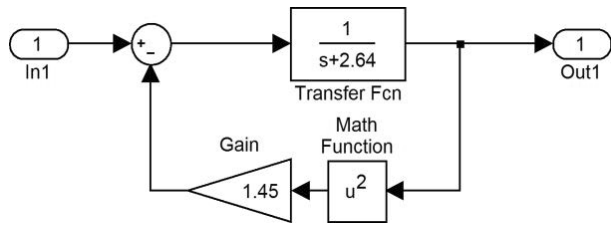


Fig 5. Simulink-model of the test system

Analytical expressions of AFC and PFC for the first, second and third order model were received:

$$|W_1(j\omega)| = \frac{1}{\sqrt{\alpha^2 + \omega^2}}, \quad \arg W_1(j\omega) = -\arctg \frac{\omega}{\alpha};$$

$$|W_2(j\omega_1, j\omega_2)| = \frac{\beta}{\sqrt{(\alpha^2 + \omega_1^2)(\alpha^2 + \omega_2^2)[\alpha^2 + (\omega_1 + \omega_2)^2]}},$$

$$\arg W_2(j\omega_1, j\omega_2) = -\arctg \frac{(2\alpha^2 - \omega_1\omega_2)(\omega_1 + \omega_2)}{\alpha(\alpha^2 - \omega_1\omega_2) - \alpha(\omega_1 + \omega_2)^2};$$

$$|W_3(j\omega_1, j\omega_2, j\omega_3)| = \sqrt{[\operatorname{Re}(W_3(j\omega_1, j\omega_2, j\omega_3))]^2 + [\operatorname{Im}(W_3(j\omega_1, j\omega_2, j\omega_3))]^2} = \frac{2\beta^2}{3} \frac{1}{\sqrt{[\alpha^2 + (\omega_1 + \omega_2 + \omega_3)^2][\alpha^2 + \omega_1^2][\alpha^2 + \omega_2^2][\alpha^2 + \omega_3^2]}} \times \frac{\sqrt{[3\alpha^2 - (\omega_1 + \omega_2)(\omega_1 + \omega_2) - (\omega_1 + \omega_2)(\omega_2 + \omega_3) - (\omega_1 + \omega_3)(\omega_2 + \omega_3)]^2 + 16\alpha^2(\omega_1 + \omega_2 + \omega_3)^2}}{\sqrt{[\alpha^2 + (\omega_1 + \omega_2)^2][\alpha^2 + (\omega_1 + \omega_3)^2][\alpha^2 + (\omega_2 + \omega_3)^2]}}$$

$$\arg W_3(j\omega_1, j\omega_2, j\omega_3) = \arctg \frac{\operatorname{Im} W_3(j\omega_1, j\omega_2, j\omega_3)}{\operatorname{Re} W_3(j\omega_1, j\omega_2, j\omega_3)} = -\arctg \frac{DA - CB}{AB + CD},$$

where

$$A = 3\alpha^2 - 3\omega_1\omega_2 - 3\omega_2\omega_3 - 3\omega_1\omega_3 - \omega_1^2 - \omega_2^2 - \omega_3^2; \quad B = uw - v\tau;$$

$$C = 4\alpha(\omega_1 + \omega_2 + \omega_3); \quad D = vw + uz;$$

$$u = \alpha^3 - \alpha\omega_1\omega_2 - \alpha\omega_2\omega_3 - \alpha(\omega_1 + \omega_2 + \omega_3); \quad v = (\omega_1 + \omega_2 + \omega_3)(2\alpha^2 - \omega_1\omega_2 - \omega_2\omega_3);$$

$$w = (\alpha^2 - \omega_1\omega_2 - \omega_2\omega_3)(\alpha^2 - \omega_1\omega_2 - \omega_1\omega_3) - \alpha^2(\omega_1 + \omega_2 + \omega_3)^2;$$

$$z = \alpha(\omega_1 + \omega_2 + \omega_3)(2\alpha^2 - 2\omega_1\omega_2 - \omega_1\omega_3 - \omega_2\omega_3).$$

The main purpose was to identify the multi-frequency performances characterizing nonlinear and dynamical properties of nonlinear test system [11–21]. Volterra model in the form of the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> order polynomial is used. Thus, test system properties are characterized by transfer functions of

$W_1(j\omega)$ ,  $W_2(j\omega_1, j\omega_2)$ ,  $W_3(j\omega_1, j\omega_2, j\omega_3)$  – by Fourier-images of weight functions  $w_1(t)$ ,  $w_2(t_1, t_2)$  and  $w_3(t_1, t_2, t_3)$ .

Structure charts of identification procedure – determinations of the 1<sup>th</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> order AFC of CC are presented accordingly in fig. 6–8.

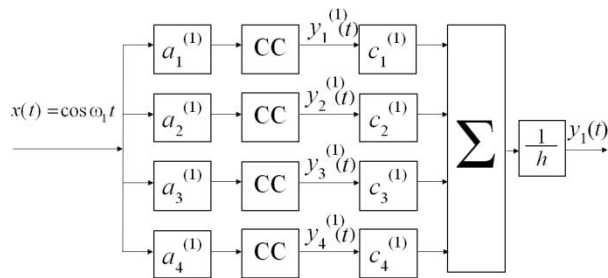
The weighted sum is formed from received signals – responses of each group from fig. 6–8. As a result the partial components of CC responses  $y_1(t)$ ,  $y_2(t)$  and  $y_3(t)$  are got. For each partial component of response the Fourier transform (the FFT is used) is calculated, and from received spectrum only an informative harmonics (which amplitudes represent values of required characteristics of the first, second and third orders AFC) are taken.

The first order AFC  $|W_1(j\omega)|$  and PFC  $\arg W_1(j\omega)$  is received by extracting the harmonics with frequency  $\omega$  from the spectrum of the CC partial response  $y_1(t)$  to the test signal  $x(t)=(A/2)\cos(\omega t)$ .

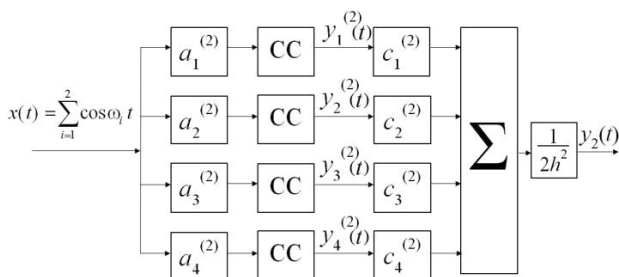
The second order AFC  $|W_2(j\omega, j(\omega+\Omega_1))|$  and PFC  $\arg W_2(j\omega, j(\omega+\Omega_1))$  having  $\omega_1=\omega$  and  $\omega_2=\omega+\Omega_1$  were received by extracting the harmonics with summary frequency  $\omega_1+\omega_2$  from the spectrum of the CC partial response  $y_2(t)$  to the test signal  $x(t)=(A/2)(\cos(\omega_1 t)+\cos(\omega_2 t))$ .

The third order AFC  $|W_3(j\omega, j(\omega+\Omega_1), j(\omega+\Omega_2))|$  and PFC  $\arg W_3(j\omega, j(\omega+\Omega_1), j(\omega+\Omega_2))$  having  $\omega_1=\omega$ ,  $\omega_2=\omega+\Omega_1$  and  $\omega_3=\omega+\Omega_2$ , were received by extracting the harmonics with summary frequency  $\omega_1+\omega_2+\omega_3$  from the spectrum of the CC partial response  $y_3(t)$  to the test signal  $x(t)=(A/2)(\cos\omega_1 t+\cos\omega_2 t+\cos\omega_3 t)$ .

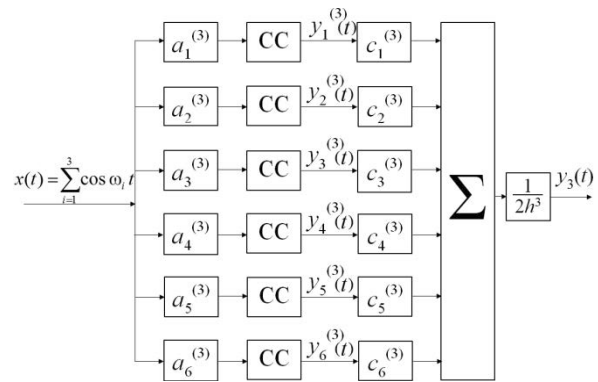
The results (first, second and third order AFC and PFC which had been received after procedure of identification) are represented in fig. 9–11.



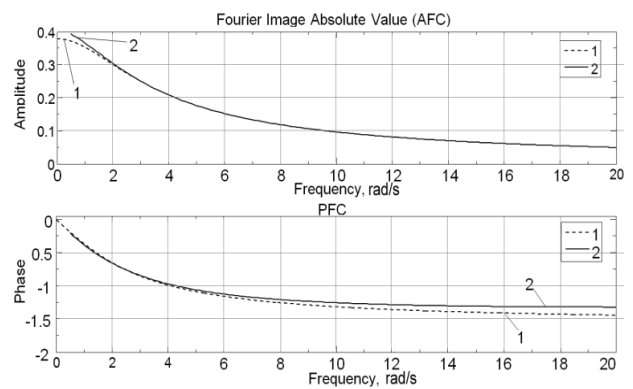
**Fig 6. The structure chart of identification using 1st order Volterra model in frequency domain, number of experiments  $N=4$ :  $a_1=-2h$ ,  $a_2=-h$ ,  $a_3=h$ ,  $a_4=2h$ ;  $c_1=-1/12$ ,  $c_2=-2/3$ ,  $c_3=2/3$ ,  $c_4=1/12$**



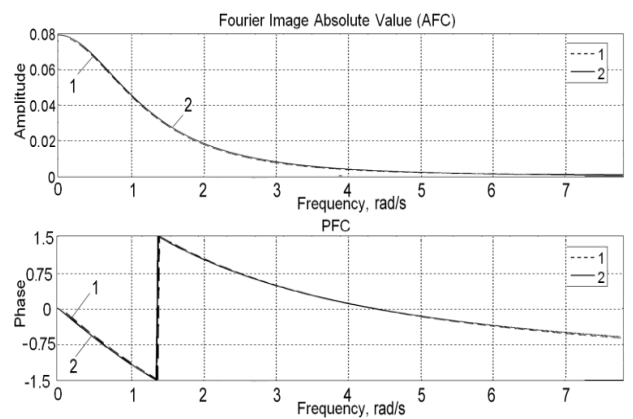
**Fig. 7. The structure chart of identification using 2nd order Volterra model in frequency domain, number of experiments  $N=4$ :  $a_1=-2h$ ,  $a_2=-h$ ,  $a_3=h$ ,  $a_4=2h$ ;  $c_1=-1/12$ ,  $c_2=4/3$ ,  $c_3=4/3$ ,  $c_4=-1/12$**



**Fig. 8. The structure chart of identification using 3rd order Volterra model in frequency domain, number of experiments  $N=6$ :  $a_1=-3h$ ,  $a_2=-2h$ ,  $a_3=-h$ ,  $a_4=h$ ,  $a_5=2h$ ,  $a_6=3h$ ;  $c_1=-1/8$ ,  $c_2=-1$ ,  $c_3=13/8$ ,  $c_4=-13/8$ ,  $c_5=1$ ,  $c_6=1/8$**



**Fig. 9. First order AFC and PFC of the test system: analytically calculated values (1), section estimation values with number of experiments for the model  $N=4$  (2)**



**Fig. 10. Second order AFC and PFC of the test system: analytically calculated values (1), sub-diagonal cross-section values with number of experiments for the model  $N=4$  (2),  $\Omega_1=0,01$  rad/s**

Numerical values of identification accuracy using interpolation method for the test system are represented in table 3, where:  $n$  – order of the estimated Volterra kernel,  $N$  – approximation order/number of interpolation knots (number of experiments).

Table 5

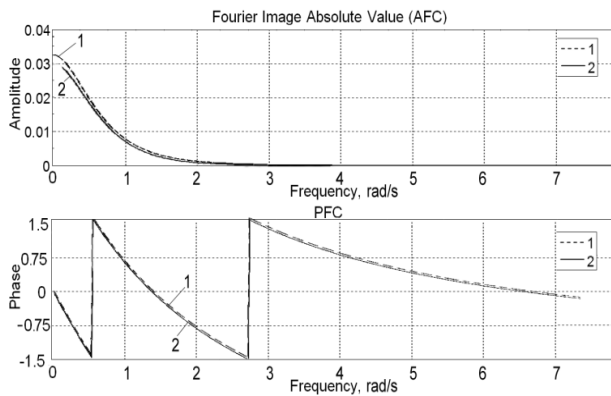
**Standard deviation for interpolation method with noise impact (bold font shows the best values)**

n	N	Noise level = 10%		Noise level = 1%		Improvement	
		SD for AFC	SD for PFC	SD for AFC	SD for PFC	for AFC, times	for PFC, times
(without / with denoising)							
1	2	0,000097 / 0,000063	0,09031 / 0,07541	-	-	1,540	1,198
	4	0,000271 / 0,000181	0,07804 / 0,06433	-	-	1,497	1,213
	6	0,000312 / 0,000223	0,12913 / 0,09889	-	-	1,399	1,306
2	2	0,000920 / 0,000670	0,52063 / 0,51465	-	-	1,373	1,012
	4	0,001972 / 0,001663	0,28004 / 0,06877	-	-	1,186	4,072
	6	0,004165 / 0,003908	0,39260 / 0,19237	-	-	1,066	2,041
3	4	-	-	0,000288 / 0,000288	0,89857 / 0,61251	1,003	1,467
	6	-	-	0,000461 / 0,000352	0,84868 / 0,59319	1,310	1,431

Table 3

**Numerical values of identification accuracy using interpolation method**

n	N	AFC relative error, %	PFC relative error, %
1	2	2,1359	2,5420
	4	0,3468	2,0618
	6	0,2957	1,9311
2	2	30,284	76,822
	4	2,0452	3,7603
	6	89,209	5,9438
3	4	10,981	1,628
	6	10,764	1,5522

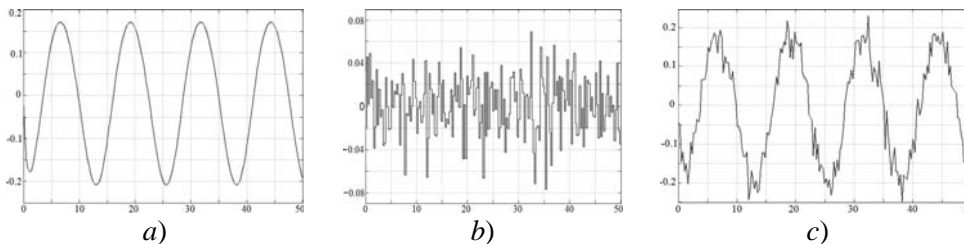


**Fig. 11. Third order AFC and PFC of the test system: analytically calculated values (1), sub-diagonal cross-section values with number of experiments for the model  $N=6$  (2),  $\Omega_1=0,01$  rad/s,  $\Omega_2=0,1$  rad/s**

Comparison of the numerical values for identification accuracy using interpolation method [22–23] and approximation one [21; 25; 42; 46] for the test system is presented in table 4.

**4. The study of noise immunity of the identification method**

Experimental researches of the noise immunity of the identification method were made. The main purpose was the studying of the noise impact (noise means the inexactness of the measurements) to the characteristics of the test system model using interpolation method in frequency domain.



**Fig. 12, a – Test signal and; b – Random noise with 25% amplitude of test signal; c – The “noised” signal of the test system**



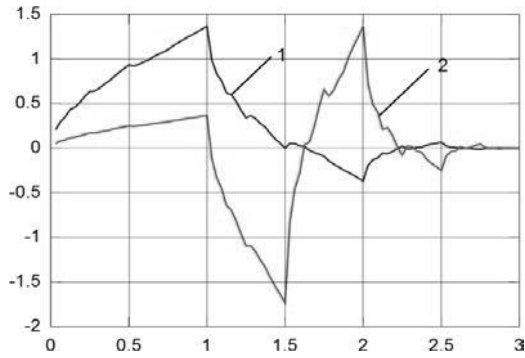


Fig. 13. The Daubechies 2nd level scaling (1) and wavelet (2) functions

After that procedure the Random Noise signal (with the form shown in fig. 12, *b*) where added to the test system output signal. This steps where performed to simulate inexactness of the measurements in the model. The sum of these two signals for the linear test model signal is shown in fig. 12, *c*.

Table 4  
Identification accuracy using approximation and interpolation methods

n	N	AFC relative error, %		PFC relative error, %	
		Approximation	Interpolation	Approximation	Interpolation
1	2	3,6429	2,1359	3,3451	2,5420
	4	1,1086	0,3468	3,1531	2,0618
	6	0,8679	0,2957	3,1032	1,9311
2	2	26,0092	30,2842	30,2842	76,8221
	4	3,4447	2,0452	2,0452	3,7603
	6	7,3030	89,2099	4,6408	5,9438
3	4	72,4950	10,981	10,9810	1,628
	6	74,4204	10,7642	10,7642	1,5522

The simulations with the test model were performed. Different noise levels were defined for different order of the Volterra model.

The adaptive wavelet denoising was used to reduce the noise impact on final characteristics of the

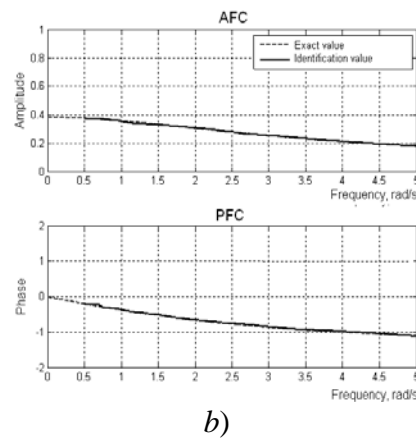
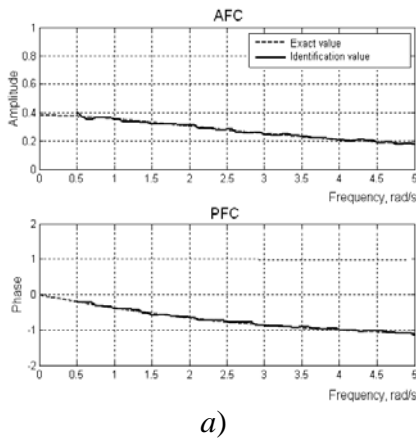


Fig. 14. Noised (a) and denoised (b) characteristics (AFC – top, PFC – bottom) of the 1st order model of the test system with level of noise 25%

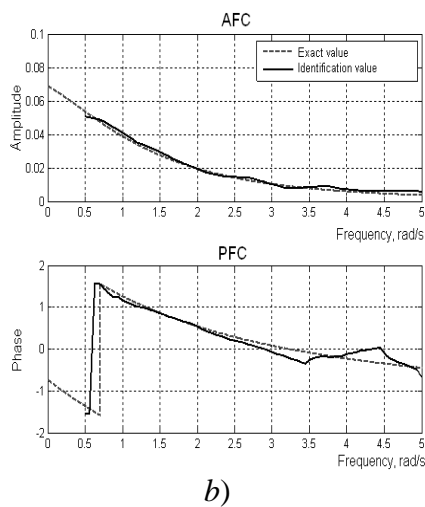
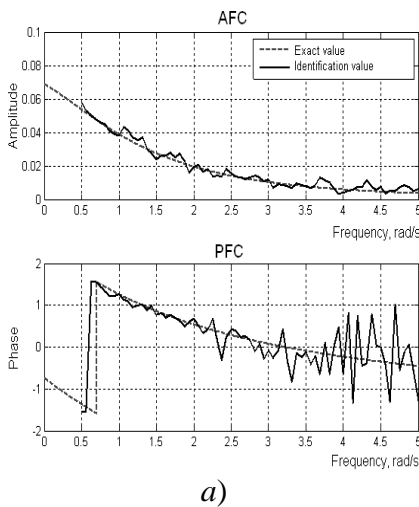


Fig. 15. Noised (a) and denoised (b) characteristics (AFC – top, PFC – bottom) of the 2nd order model of the test system with level of noise 10%

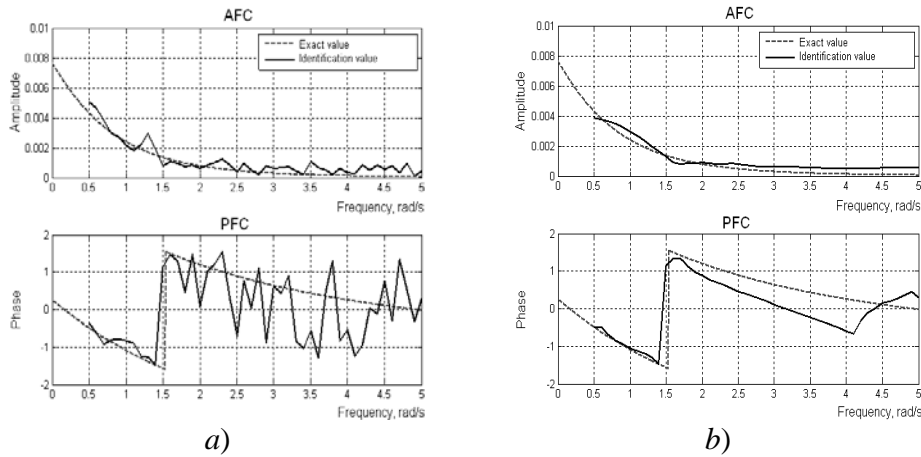


Fig. 16. Noised (a) and denoised (b) characteristics (AFC – top, PFC – bottom) of the 3rd order model of the test system with level of noise 1%

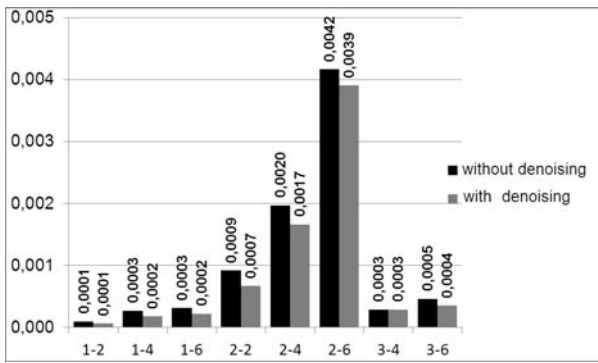


Fig. 17. Standard deviation changing for AFC using adaptive Wavelet-denoising

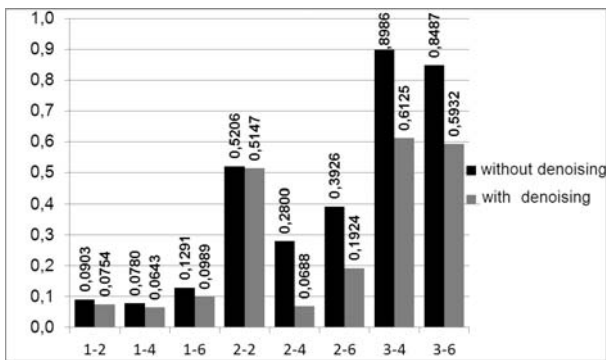


Fig. 18. Standard deviation changing for PFC using adaptive Wavelet-denoising

test system [55–60]. The Daubechie wavelet of the 2 and 3 level was chosen (fig. 13) and used for the AFC and PFC denoising respectively.

The first order (linear) model was tested with the level of noise 25% and 10% and showed excellent level of noise immunity. The noised (fig. 14, a) and de-noised (filtered) (Fig. 14, b) characteristics (AFC and PFC) with level of noise 25% are presented.

The second order (nonlinear) model was tested with the level of noise 10% and 1% and showed good level of noise immunity. The noised (fig.15, a) and de-noised (filtered) (fig. 15, b) characteristics (AFC and PFC) with level of noise 10% are presented.

The third order (nonlinear) model was tested with the level of noise 10% and 1% and showed good level of noise immunity. The noised (fig. 16, a) and de-noised (filtered) (fig.16, b) characteristics (AFC and PFC) with level of noise 1% are presented.

The numerical values of standard deviation (SD) of the identification accuracy before and after wavelet denoising procedure are presented in Table 5.

The diagrams showing the improvement of standard deviation for identification accuracy using the adaptive wavelet denoising of the received characteristics (AFC and PFC) are shown in fig. 17 and fig. 18 respectively [61–62].

### 5. Engineering software tools

The identification methodology is implemented with approximation and interpolation methods. The structured scheme of the computational process of the identification procedure is shown in the fig. 19 [63–65].

The hardware platform of the experimental researches using developed software consists of IBM–PC compatible computer with two soundcards (Fig. 20). This allows characterizing the final results as reliable ones. Onboard soundcards (motherboard built-in) has much worst characteristics and higher unevenness of its AFC. Thus, it cannot be used in experimental researches.

The toolkit is organized from software in Matlab language. The first part is assigned for test signals generation with minimal impact of the operation system of used PC. The experimental results processing is implemented in the second part. The 13 modules were developed dur-

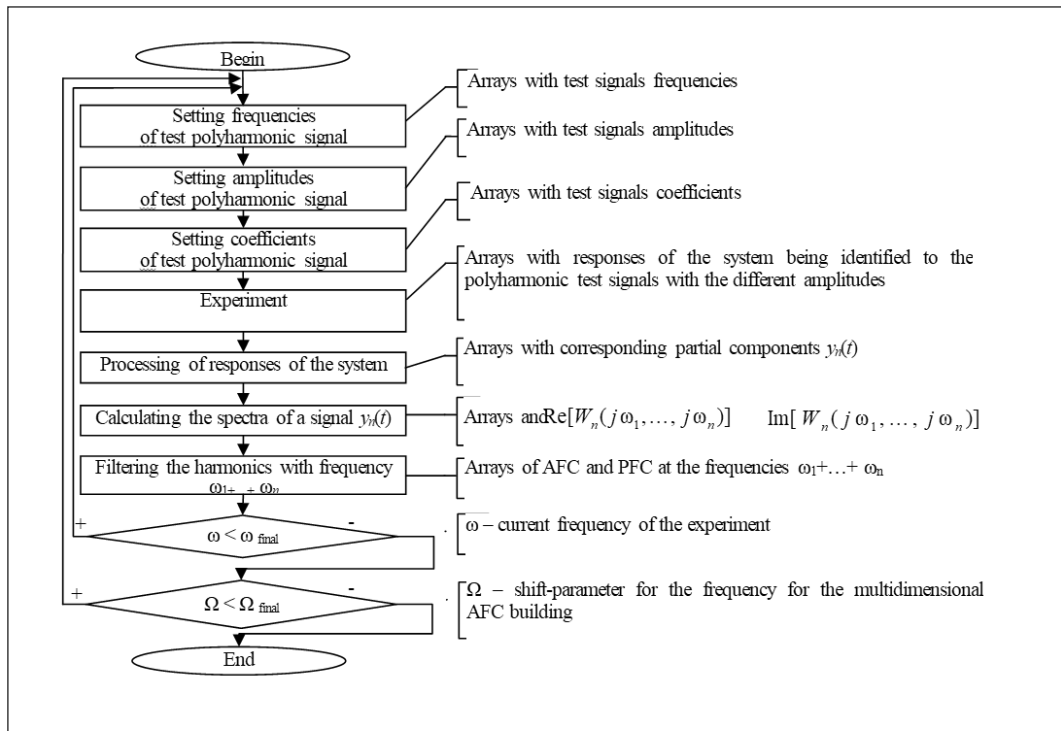


Fig. 19. Structured scheme of the computational process of the identification procedure

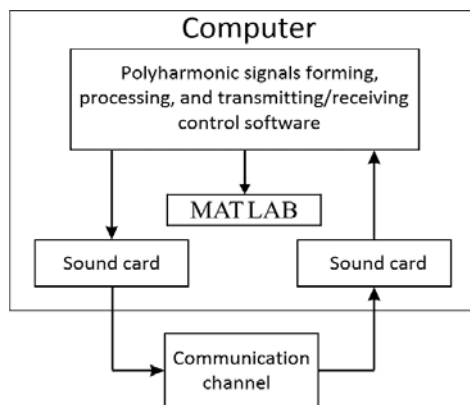


Fig. 20. The general scheme of the experiment

ing the software part implementation. The structured scheme of the software toolkit for nonlinear dynamical systems identification is shown in fig. 21. The implemented in module *main\_gui.fig* visual interface contains all visual components of the windows application used to control the parameters of the experiment (fig. 22).

An initialization of the identification process parameters of the nonlinear system being identified is performed in the main module (*main\_form.m*). The list of those parameters:

- 1) start frequency  $f_1$  of harmonic signals for the experiment;
- 2) quantity of the experimental steps (defines the quantity of the subdiagonal sections of the AFC and the final frequency of the experiment);

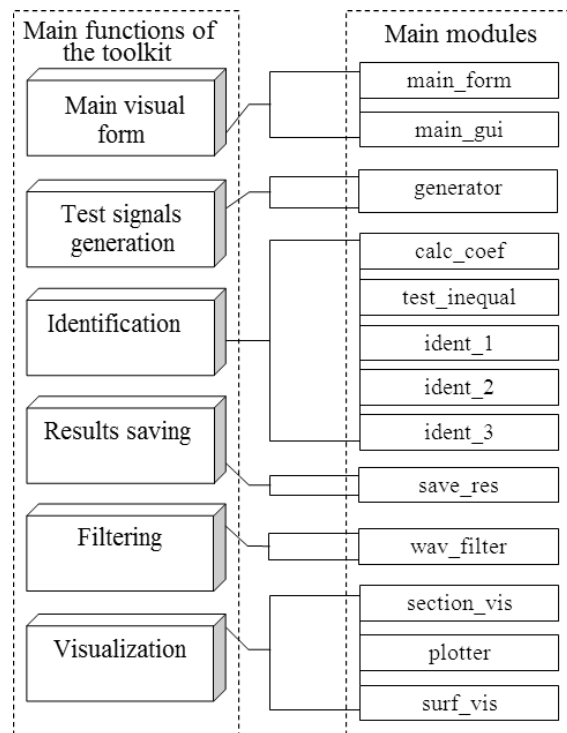


Fig. 21. The structured scheme of the software for nonlinear dynamical systems identification

- 3) polyharmonic test frequency step;
- 4) quantity of the experiments repeats that allows to average results received for current nonlinear object;
- 5) quantity of the signal samples being sent to the

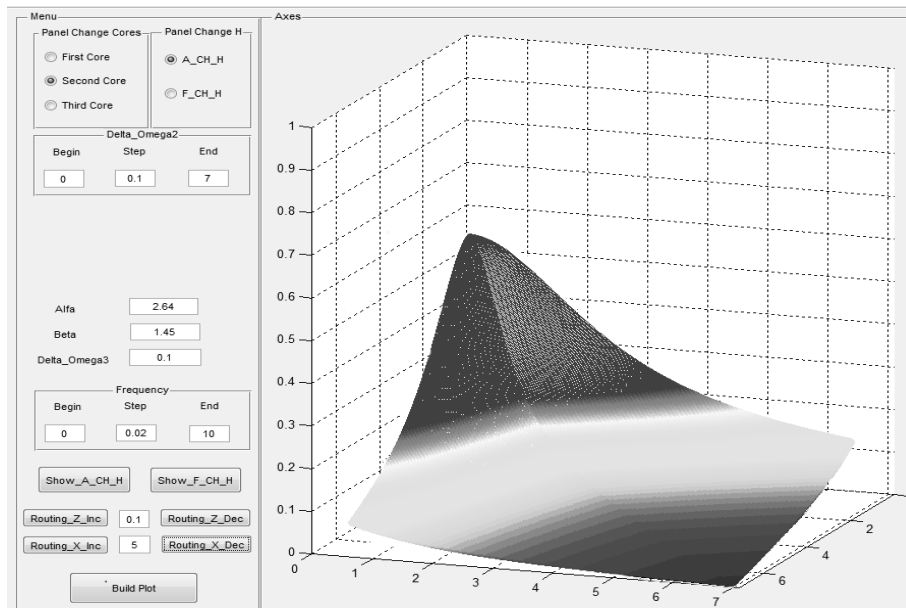


Fig. 22. Part of the main window of the software used for standard characteristics constructing (subdiagonal sections and 3D surfaces)

nonlinear dynamical system input (this number have to be divisible by 2 to obtain correct work on the Fast Fourier Transform);

6) sampling frequency of the sound signal being sent to the nonlinear dynamical system input;

7) kernel order for the Volterra model;

8) approximation order/experiments quantity for the approximation/interpolation method of the coefficients calculation and experiments providing;

9) shift between the frequencies  $f_2$  and  $f_1$  for the polyharmonic test signals (for nonlinear models);

10) shift between the frequencies  $f_3$  and  $f_2$  for the polyharmonic test signals (for nonlinear models);

11) amplitudes array for the mono- or polyharmonic test signals (depends on model order);

12) corresponding coefficients array calculated using amplitudes of the test signals;

13) time array for sound test signal forming;

14) response array (consists of the values of tested nonlinear dynamical system responses to the test harmonic signal);

15) AFC array (consists AFC points values of tested nonlinear dynamical system).

The coefficients for identification during the forming the test signals are depending on its amplitudes. The calculation of such coefficients is performed by the matrix method of SLAE decision and it's performed in *calc\_coef* module. One of the identification modules of the chosen order (*ident\_1*, *ident\_2*, *ident\_3*) begins working after starting the identification process. In common way:

– the test harmonic signals with selected amplitudes (depending on method) are generated;

– generated signals data arrays are sent to the input of the nonlinear system;

– the signals received after nonlinear system has the visual form shown in fig. 23 and they have to be preprocessed (multiplication the signals and output coefficients);

– the total (sum) response of all test signals is calculated;

– the points of AFC array for the responses of the system being identified are calculated;

– the data received during identification at the current frequency step is saved to the file.

The identification method is organized using the approximation and interpolation method [62]. The identification of the nonlinear system of the 1<sup>st</sup> order is presented as textual algorithm:

*Step 1.* The main module (*main\_form.m*) sending the parameters of the identification to the module *ident\_1.m*.

*Step 2.* All supporting local variables and arrays are initialized.

*Step 3.* The loop for experiment repetitions with selected iterations.

*Step 3.1.* The loop for changing the test frequency from start value with selected step for the selected quantity of steps (specified in parameters of the identification).

*Step 3.1.1.* The  $f_1$  frequency increment at current experiment step.

*Step 3.1.2.* The test harmonic signal generating.

*Step 3.1.3.* Searching for the  $f_1$  frequency position in signal spectra.

*Step 3.1.4.* The loop for each test signal amplitude in the experiment.

*Step 3.1.4.1.* The signal with specified amplitude is forming.

*Step 3.1.4.2.* The formed signal is sending to the output of the soundcard.

*Step 3.1.4.3.* The signal is receiving from the input of the soundcard.

*Step 3.1.4.4.* The multiplication of the received signal data (response) with specified amplitudes by corresponding coefficients.

*Step 3.1.4.5.* The total response calculation by summation of received signals groups.

*Step 3.1.4.6.* If the selected method is interpolation then performing division of the total response by additional method coefficient.

*Step 3.1.4.7.* Calculating the signal spectra using Fast Fourier Transform.

*Step 3.1.4.8.* Calculating the value of informational harmonics with position detected at the *Step 3.1.3.*

*Step 3.1.4.9.* Calculating the AFC and amplitude correction.

*Step 3.1.5.* Saving the experimental data in files as arrays with identification results for the frequency  $f_1$ .

To save the data of the identification results for subsequent system model in frequency domain building the m-files are used. The names of those files are fully showing the parameters of the experiment. The files are saved at the Results subfolder of the program folder.

The format of file name with identification data of the test model allowing cataloging results looks as  $Vn(N)_{Na\_metb\_nc\_mad\_mke\_w1\_f-g-h\_dw1\_i\_w2\_j}$ .mat and consists of such fields:

$n$  – model (Volterra kernel) order;

$N$  – approximation order;

$a$  – discretization order;

$b$  – method (1 – interpolation, 0 – approximation);

$c$  – noise level (in % relatively to test signal level);

$d$  – scaling the test signals amplitudes relatively to initial (in %);

$e$  – scaling the test signals coefficients relatively to initial (in %);

$f$  – start frequency  $f_1$  value;

$g$  – step of the frequency  $f_1$  changing;

$h$  – final value of the frequency  $f_1$ ;

$i$  – shifting between the frequencies  $f_2$  and  $f_1$ ;

$j$  – the value of the frequency  $f_3$ .

The format of file name with identification data of the real communication channel allowing cataloging results looks as  $Final\_xxxx\_yyyy\_zzzz\_Vk-N\_mm-dd-yy\_HH-MM-SS$ .mat and consists of such fields:

$xxxx$  – start frequency of the current experiment (section);

$yyy$  – shift between the frequencies  $f_2$  and  $f_1$ ;

$zzz$  – the value of the frequency  $f_3$ ;

$n$  – model (Volterra kernel) order;

$N$  – approximation order;

$mm-dd-yy\_HH-MM-SS$  – current date and time in selected format.

On the results of the identification data contained in the files, it is possible to draw two-dimensional plots (module *sections\_vis.m*) – subdiagonal AFC sections of the nonlinear dynamical system. A 3-dimensional plots (modules *surf\_vis.m*, *plotter.m*) – surfaces are built of the subdiagonal AFC sections of the nonlinear dynamical system by changing shifting between the frequencies  $f_2$  and  $f_1$  and changing frequency  $f_3$  for the 2<sup>nd</sup> and 3<sup>rd</sup> order models respectively.

## 6. The technique and hardware-software tools of radiofrequency CC identification

Experimental research of the Ultra High Frequency range CC were done [66]. The main purpose was the identification of multi-frequency characteristics that characterize nonlinear and dynamical properties of the CC. Volterra model in the form of the second order polynomial is used. Thus physical CC properties are characterized by transfer functions of  $W_1(j2\pi f)$ ,  $W_2(j2\pi f_1, j2\pi f_2)$  and  $W_3(j2\pi f_1, j2\pi f_2, j2\pi f_3)$  – by the Fourier-images of weighting functions  $w_1(t)$ ,  $w_2(t_1, t_2)$  and  $w_3(t_1, t_2, t_3)$ .

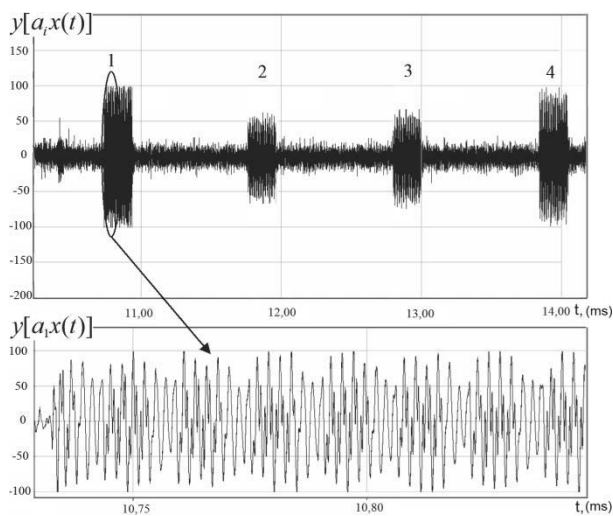
Implementation of identification method on the IBM PC computer basis has been carried out using the developed software in Matlab software. The software allows automating the process of the test signals forming with the given parameters (amplitudes and frequencies). Also this software allows transmitting and receiving signals through an output and input section of PC soundcard, to produce segmentation of a file with the responses to the fragments, corresponding to the CC responses being researched on test polyharmonic effects with different amplitudes.

In experimental research two identical marine transceivers S.PRADIO A/S SAILOR RT2048 VHF (the range of operational frequencies is 154,4–163,75 MHz) and IBM PC with Creative Audigy 4 soundcards were used. Sequentially AFC of the first and second orders were defined. The method of identification with number of experiments  $N=4$  was applied. Structure charts of identification procedure – determinations of the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> order AFC of CC are presented accordingly in fig. 6 – fig. 8.

The CC received responses  $y[a,x(t)]$  to the test signals  $a,x(t)$ , compose a group of the signals, which amount is equal to the used number of experiments  $N$  ( $N=4$ ), shown in fig. 23. In each following group the signals frequency increases by magnitude of cho-

sen step. A cross-correlation was used to define the beginning of each received response.

Maximum allowed amplitude in described experiment with use of sound card was  $A=0,25V$  (defined experimentally). The range of frequencies was defined by the sound card pass band (20...20000 Hz), and frequencies of the test signals has been chosen from this range, taking into account restrictions specified above. Such parameters were chosen for the experiment: start frequency  $f_s=125$  Hz; final frequency  $f_c=3125$  Hz; a frequency change step  $F=125$  Hz; to define AFC of the second order determination, an offset on frequency  $F_1=f_2-f_1$  was increasingly growing from 201 to 3401 Hz with step 100 Hz.



**Fig. 23. The group of signals received from CC with amplitudes: -1 (1); -1/2 (2); 1/2 (3); 1 (4); N=4**

Maximum allowed amplitude in described experiment with use of sound card was  $A=0,25V$  (defined experimentally). The range of frequencies was defined by the sound card pass band (20...20000 Hz), and frequencies of the test signals has been chosen from this range, taking into account restrictions specified above. Such parameters were chosen for the experiment: start frequency  $f_s=125$  Hz; final frequency  $f_c=3125$  Hz; a frequency change step  $F=125$  Hz; to define AFC of the second order determination, an offset on frequency  $F_1=f_2-f_1$  was increasingly growing from 201 to 3401 Hz with step 100 Hz.

The weighed sum is formed from received signals – responses of each group (fig. 6, fig. 7). As a result we get partial components of the response of the CC  $y_1(t)$  and  $y_2(t)$ . For each partial component of the response a Fourier transform (the Fast Fourier Transform is used) is calculated. Only informative harmonics (which amplitudes represents values of required characteristics of the first, second and third order AFC) are taken from received spectrum.

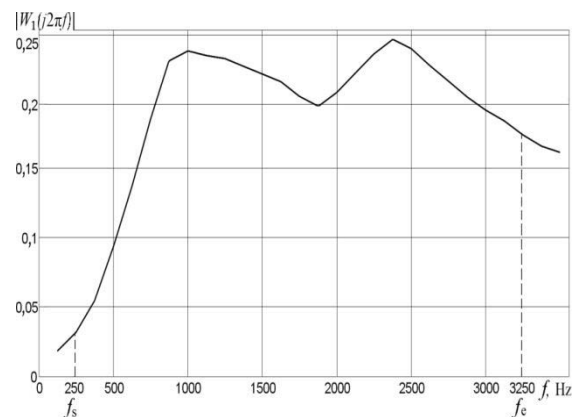
The first order amplitude-frequency characteristic  $|W_1(j2\pi f)|$  is received by extracting the harmonics with frequency  $f$  from the spectrum of the partial response of the CC  $y_1(t)$  to the test signal  $x(t)=(A/2)\cos 2\pi f t$ .

The second order AFC  $|W_2(j2\pi f_1, j2\pi f_2)|$ , where  $f_1=f$  and  $f_2=f+F_1$  was received by extracting the harmonics with summary frequency  $f_1+f_2$  from the spectrum of the partial response of the CC  $y_2(t)$  to the test signal  $x(t)=(A/2)(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$ .

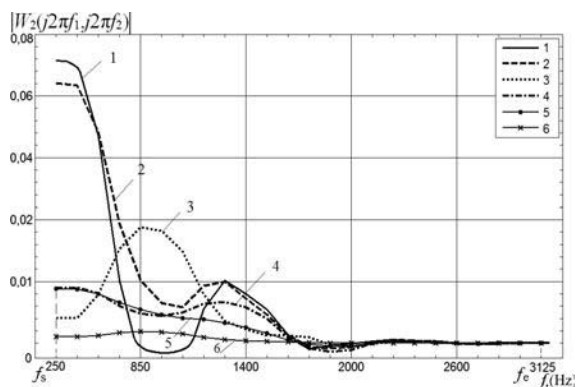
The third order AFC  $|W_3(j2\pi f_1, j2\pi f_2, j2\pi f_3)|$ , where  $f_1=f, f_2=f+F_1$  and  $f_3=127,5$  Hz were received by extracting the harmonics with summary frequency  $f_1+f_2+f_3$  from the spectrum of the partial response of the CC  $y_3(t)$  to the test signal  $x(t)=(A/2)(\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_3 t)$  (fig. 8).

The wavelet noise-suppression was used to smooth the output data of the experiment [9]. The results received after digital data processing of the data of experiments (wavelet “Coiflet” denoising) for the first, second and third order AFC are presented in fig. 24 – fig. 27.

The surfaces shown in fig. 26 – fig. 27 are built from sub-diagonal sections that are received separately. We used  $F_1$  as growing parameter of identification with different value for each section.



**Fig. 24. AFC of the first order after wavelet “Coiflet” 2nd level noise-suppression**



**Fig. 25. Subdiagonal sections of AFCs of the second order after wavelet “Coiflet” 2nd level noise-suppression at different frequencies  $F_1$ : 201 (1), 401 (2), 601 (3), 801 (4), 1001 (5), 1401 (6) Hz**

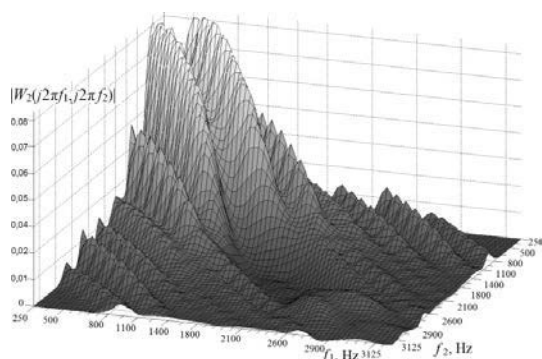


Fig. 26. Surface built of AFCs of the second order after wavelet “Coiflet” 3rd level noise-suppression

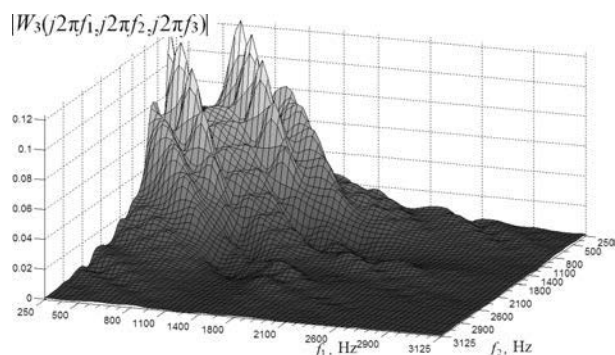


Fig. 27. Surface built of AFCs of the third order after wavelet “Coiflet” 3rd level noise-suppression, where  $f_3=127,5$  Hz

### Conclusions

Communication channel as a media for remote sensing systems functioning is analyzed. Nonlinear effects of the environments have great impact

on result data received in experiments. The method based on Volterra model using polyharmonic test signals for identification nonlinear dynamical systems is analyzed. To differentiate the responses of system for partial components we use the method based on linear combination of responses on test signals with different amplitudes.

New values of test signals amplitudes were defined and they are greatly raising the accuracy of identification compared to amplitudes and coefficients written in [16]. The accuracy of identification of nonlinear part of the test system growth 2 times and the standard deviation in this case is about 5%.

The interpolation method of identification using the hardware methodology written in [62] is applied for construction of informational Volterra model as an APC of the first and second order for UHF band radio channel.

Received results reveal essential nonlinearity of the CC that leads to distortions of signals in radio broadcasting devices, reduces the important indicators of the TCS: accuracy of signals reproduction, throughput, noise immunity.

The noise immunity is very high for the linear model, high enough for the second order nonlinear model and has moderate noise immunity for the third order model. The wavelet denoising is very effective and gives the possibility to improve the quality of identification of the noisy measurements up to 1,54 and 4,07 times for the AFC and PFC respectively.

Final characteristics of the CC have to be used to maintain sensor systems to improve the adequateness of received data.

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### **МЕТОДИ ТА ІНСТРУМЕНТАЛЬНІ ЗАСОБИ ІДЕНТИФІКАЦІЇ НЕЛІНІЙНИХ ДИНАМІЧНИХ СИСТЕМ НА ОСНОВІ МОДЕЛЕЙ ВОЛЬТЕРРИ В ЧАСТОТНІЙ ОБЛАСТІ**

*Досліджується точність та обчислювальна стійкість методів детермінованої ідентифікації нелінійних динамічних систем у вигляді багатовимірних амплітудно- і фазочастотних характеристик. Розглядаються апроксимаційний та інтерполяційний методи ідентифікації з використанням тестових полігармонічних сигналів. Обчислювальна стійкість процедури ідентифікації забезпечується за допомогою методу регуляризації некоректних задач. Для згладжування оцінок отриманих характеристик використовується вейвлет-фільтрація.*

**Ключові слова:** нелінійні динамічні системи, непараметрична ідентифікація, модель Вольтерри, багатовимірні частотні характеристики, полігармонічні сигнали, регуляризація, вейвлет-фільтрація.

### **МЕТОДЫ И ИНСТРУМЕНТАЛЬНЫЕ СРЕДСТВА ИДЕНТИФИКАЦИИ НЕЛИНЕЙНЫХ ДИНАМИЧЕСКИХ СИСТЕМ НА ОСНОВЕ МОДЕЛЕЙ ВОЛЬТЕРРЫ В ЧАСТОТНОЙ ОБЛАСТИ**

*Исследуется точность и вычислительная устойчивость методов детерминированной идентификации нелинейных динамических систем в виде многомерных амплитудно- и фазочастотных характеристик. Рассматриваются аппроксимационный и интерполяционный методы идентификации с использованием тестовых полигармонических сигналов. Вычислительная устойчивость процедуры идентификации обеспечивается с помощью метода регуляризации некорректных задач. Для сглаживания оценок полученных характеристик используется вейвлет-фильтрация.*

**Ключевые слова:** нелинейные динамические системы, непараметрическая идентификация, модель Вольтерры, многомерные частотные характеристики, полигармонические сигналы, регуляризация, вейвлет-фильтрация.